

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2018-21]

B.A./B.Sc. FIRST SEMESTER (July – December) 2018

Mid-Semester Examination, September 2018

Date : 27/09/2018

MATHEMATICS (General)

Time : 12 noon – 1 pm

Paper : I

Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

Answer **any four** from question nos. 1 to 6 :

[4×4]

1. Show that the product of all the four values of $(1+i\sqrt{3})^{3/4}$ is 8.

2. Show that the roots of the equation

$$\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = \frac{1}{x} \quad (a > b > c > 0) \text{ are all real.}$$

3. Solve, $x^3 + 12x - 12 = 0$ by Cardan's Method.

4. Express in factors : $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

5. Solve using matrix inversion :

$$x+y+z = 4$$

$$2x-y+3z = 1$$

$$3x+2y-z = 1$$

6. Show that $x+y+z = 6$

$$x+2y+3z=14$$

$$x+4y+7z=30$$

is consistent and solve them.

[1+3]

Group – B

Answer **any one** from question nos. 7 & 8 :

[1×5]

7. a) Show that $\sqrt{3}$ is irrational.

b) Given that $x > 0$, show that there is an irrational number y such that $0 < y < x$.

[3+2]

8. a) Find $\lim_{x \rightarrow 2} (x-2) \sin \left(\frac{1}{x-2} \right)$.

b) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \cos x$ is continuous.

[2+3]

Group – C

Answer **any one** from question nos. 9 & 10 :

[1×4]

9. a) Find the tangent and normal to the curve $y^2 - yx^2 - 2x^5 = 0$ at any point (x, y) on it.

[3]

b) Show that the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$ does not exist.

[1]

10. a) Evaluate the repeated limits and the double limit of $f(x,y) = x^2 \sin \frac{1}{y^2} + y^2 \sin \frac{1}{x^2}$ as $x \rightarrow 0$ and $y \rightarrow 0$. [3]
- b) Find the radius of curvature of the curve $y = 4 \sin x - \sin 2x$, at $x = \frac{\pi}{2}, y = 4$. [1]

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